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CRYSTAL OSCILLATORS` JITTER MEASUREMENTS AND ITs ESTIMATION OF PHASE NOISE

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Abstract – It is shown that common method of jitter variance estimating by integrating phase fluctuations` spectrum is not correct for sampling oscilloscope jitter measurements have been performed with synchronization to the measured signal. It is obtained that oscillators` phase noise transformation depends on the sampling oscilloscope implementation and has many options have not been investigated deep enough. The inconsistence between the phase noise transformation and measurement results interpretation lead to errors in the measurements and specifications of crystal oscillators and other high stability sources, decrease the accuracy and stability of GPS, radar-systems, telemetry systems, etc.

It is shown below that jitter variance should be estimated by integrating the spectrum of the frequency fluctuations, but not the spectrum of the phase fluctuations. It is obtained that jitter characterizes the high frequency part of the phase noise spectrum. The results obtained open the way of the phase noise evaluation from jitter measurements at frequency range up to 50 GHz and higher. The new relations between jitter measurements and phase noise are established for different types of oscilloscope synchronization measurements (transition jitter, Period jitter, Half-Period jitter, etc.).

It is obtained that famous way calculations jitter thorough Allan variance characterizes the Circle-to-Circle jitter. The methodology for jitter specification and characterization is discussed.

INTRODUCTION

As the clock speed of electronic devices has grown, the oscilloscope technique became more commonly used for jitter characterization. In oscillators` jitter measurements this method is often implemented with sampling oscilloscope synchronization to the measured signal. The main advantages of this measurement method are the following: resolution and sensitivity achieves fractions of sub pico-seconds, measurement bandwidth up to 50 GHz and higher, and reference devices are not needed. The last one is especially important for crystal oscillators (XOs) measurements, where it helps to avoid a large number of references. These advantages significantly simplify the implementation of the method and cause this technique to become widely used in recent years.

The sampling oscilloscope jitter measurement is a relatively new method of characterizing the high stability frequency sources in the time domain. This measurement method has not been studied as thoroughly as the Frequency Domain method based on the phase detector technique, and as Time Domain method based on electronic counters [1-6]. The sampling oscilloscope jitter measurement method was investigated in some papers, particular [8-20]. However, many problems have not still been treated, or some important specific these measurements have been missed.

Present work investigates the jitter and phase noise relations for different types of sampling oscilloscope measurements. In Part I the transition jitter (first jitter difference) relations with phase noise is considered. It is shown that jitter is defined mainly by white phase noise level. It is obtained that jitter measurements origins mainly from the high frequency part of the phase noise spectrum. In Part II the second jitter differences (period, circle-to-circle jitter, etc.) are investigated. The presence of the two time scales at second jitter differences is noticed. Different types of the sampling oscilloscope synchronization affects on jitter measurements are investigated.

PART I. THE TRANSITION JITTER MEASUREMENTS

For sampling oscilloscope (SO) jitter measurements with the triggering time-base from the delayed measured signal on time τ, Fig.1, we may write for signals of the trigger channel X (t) and vertical channel Y (t):

$$Y(t) = \sin\,(2\pi\nu_0\,(t+\tau) + \varphi(t+\tau))$$
$$X(t) = \sin\,(2\,\pi\nu_0 t\ + \varphi(t\,)) \qquad (1)$$

The time functions T_y (t) and T_x (t) generated of the signals (1) are [21]:

$$T_y(t) = t + \tau + \varphi(t+\tau)/2\pi\nu_0 \quad T_x(t) = t\ + \varphi(t)\,/2\pi\nu_0 \qquad (2)$$

From (2) and [21], we can write time interval error function j(t) – "the variations of the significant instants of a timing signals from their ideal positions in time" [21]:

$$j(t) = (\varphi(t+\tau) - \varphi(t))/2\pi\nu_0 \qquad (3)$$

From (3) obtain the mean square of the jitter measured by ideal sampling oscilloscope:

$$< j^2(t) > = (2/\,(2\pi\nu_0)^2\,)D_\varphi(\tau) \qquad (4)$$

here < > - designate an ensemble average, and

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$D_\varphi(\tau)$= (½) < ((φ($t+\tau$)-φ(t))^2 > is the structural function [7] (the first difference) of the phase fluctuations. The structural functions were inputted by A. Kolmogorov for turbulence processes description, and developed by A. Malahov for different types of the flicker noises present in oscillators [7]. Notice that the famous Allan variance is the first difference of averaged fractional frequency measured by electronic counter, or second difference structural function of the phase fluctuations [1-6].

The structural function implementation is effective when noise represents process with stationary first differences [7]. Consider flicker processes with spectrum dependence on frequency f^{β} where β is limited to 0 > β > –3. Formally, the processes with such spectrum have unlimited power and their spectrum approaches the infinity at $f \to 0$ (for β =1 at $f \to \infty$, also). So, these processes do not have correlation function. Otherwise, these processes have finite structural function D_φ (τ). So, they are processes with stationary first differences.

Processes with stationary first differences are more general than the stationary processes, as the former may be non-stationary [7].

Consider the relations between spectrum power density phase fluctuations and the structural functions on the example of stationary process with correlation function R (τ). As we can see from (4), $D_\varphi(\tau)$=R(0)-R(τ).

Using the relations between correlation characteristic and noise power spectrum density, we may rewrite (4) in the frequency domain

$$<j^2(t)>=\frac{8}{(2\pi\nu_0)^2}\int_0^\infty S_\varphi(f)h(f)\sin^2(\pi f\tau)df \qquad (5)$$

here $S_\varphi(f)$- power spectrum density of the phase fluctuations. Function h(f) is transfer function with equivalent cutoff frequency f_h, defined by oscillator` electronic circuits transfer function. Also (4), (5) obtained for stationary processes with finite spectrum, they will be correct for flicker type processes with β > -3. Really, at $f \to 0$ function $S_\varphi(f)$ is increasing as $1/f^{\beta}$. At the same time, the function $\sin^2(\pi f\tau)$ under integral is proportional to the f^2, and the integral at (5) has finite value at β >–3.

Equations (4), (5) represent the phase fluctuations spectrum transformation in ideal sampling oscilloscope without noise and intrinsic jitter. These equations were obtained in [8-11] and, independently, in [14-17,19,20]. In [19, 20] the jitter for various β were considered.

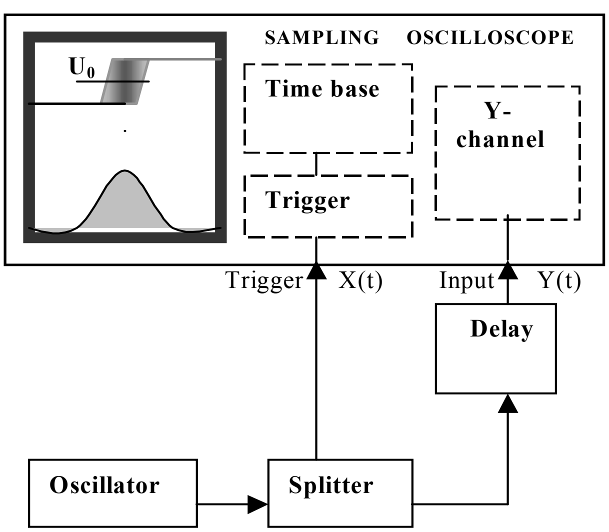


Fig. 1. The transition jitter measurements with synchronization from the measured signal.

JITTER MEASUREMENTS ESTIMATION OF PHASE NOISE

Having in mind that τ in (5) about tens and hundreds of ns, and frequency bandwidth of the phase detector F_{pd} << 1/τ, equation (5) is rewritten using the first member with f^2 of sine-function` decomposition, rewrite equation (5) in the form:

$$<j^2(t)>=\frac{2\tau^2}{\nu_0^2}\int_0^{F_{PD}} S_\varphi(f)f^2df+\frac{8}{(2\pi\nu_0)^2}\int_{F_{PD}}^{f_h} S_\varphi(f)\sin^2(\pi f\tau)df \quad (6)$$

The substitution of the relation between spectrum power density phase fluctuations $S_\varphi(f)$ and spectrum frequency fluctuations, $S_y(f) = (f^2/\nu_0^2)S_\varphi(f)$, gives:

$$<j^2(t)>=2\tau^2\{\int_0^{F_{PD}} S_y(f)df+\int_{F_{PD}}^{f_h} S_y(f)\frac{\sin^2(\pi f\tau)}{(\pi f\tau)^2}df\} \qquad (7)$$

The second integral in (7) may be interpreted as equivalent filtering of the spectrum components by low pass filter with equivalent frequency bandwidth 1/2τ. In case $f_h\tau$ <<1, (7) is simplified:

$$<j^2(t)>=2\tau^2\int_0^{f_h} S_y(f)df \qquad (8)$$

The traditional way of jitter estimation is integration S_φ(f) in the phase detector frequency range F_{PD}. From (6)–(8) follows that the min square jitter measured with self-synchronization defines by integration of the frequency fluctuations $S_y(f)$ spectrum density, but not the integration of the phase fluctuations spectrum, as usually supposed and done.

JITTER AND PHASE NOISE RELATIONS

Fig.2 is the illustration of equation (5). According to (5) and Fig.2, the phase noise is multiplied by function $4\sin^2(\pi f\tau)$. The first maximum of this function obtained at the frequency 1/(2τ). Practically, F_{pd} << 1/τ and the frequency of this maximum more than hundred times higher than typical phase detector frequency range F_{PD}. So, for jitter measurements, the influence of low frequency part of the phase noise

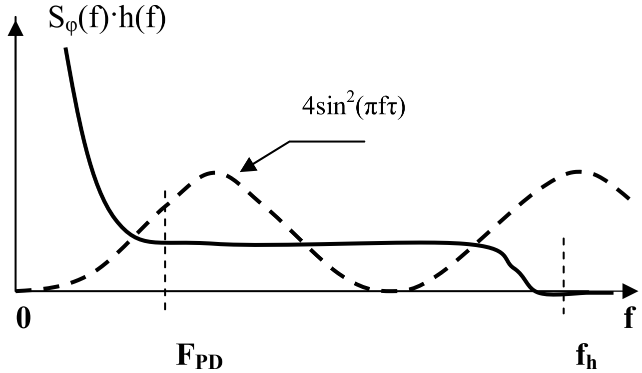


Fig. 2. The illustration the transformation of $S_\varphi(f)$ - phase fluctuations power spectrum density at sampling oscilloscope jitter measurements. F_{PD} is phase detector frequency range; $h(f)$ is the electronic circuits transfer function with equivalent cutoff frequency f_h.

spectrum\` components is reduced. As we may see, jitter measurements collect the contribution of all phase noise high frequency components above phase detector frequency range, up to cutoff frequency of the electronic circuits f_h. This information is only available from jitter measurements with this type of synchronization. More over, in phase noise measurements with phase detector technique we measure only the low frequency part of the phase noise spectrum, and do not know the phase noise behavior above the F_{PD}, shape of the h(f) function and its cutoff frequency f_h. So, the jitter measurements have very essential properties for design and oscillator noise investigations. To sum up the above:

- the contribution to the jitter value of low frequency components is significantly reduced;
- XO\` jitter measured with self synchronization is defined by White phase noise primarily;
- jitter is mainly influenced by high frequency part of the phase noise;
- jitter contains the information about h(f) function, its cutoff frequency f_h, and phase noise behavior above F_{PD};
- we may consider jitter and phase noise measurements as two distinct measurements influenced by high and low frequency parts of phase noise spectrum accordingly.

As we will see further, calculations confirm these conclusions.

NOISE COMPONENTS CONTRIBUTION TO THE JITTER VALUE

The real oscillator\` phase fluctuations power spectrum density usually is modeled by

$$S_\varphi(f) = b_{-4}\frac{1}{f^4} + b_{-3}\frac{1}{f^3} + b_{-2}\frac{1}{f^2} + b_{-1}\frac{1}{f} + b_0 \qquad (9)$$

were coefficients b_β define the levels of the different noise components. The spectrum degree β determine the slope of the single-sideband phase noise L(f) = 10 Log(0.5 $S_\varphi(f)$). Cases with β= 0, -1, -2, -3, -4 usually named respectively as White phase modulation PM (WHPM), Flicker (FLPM), Random Walk PM (RWPM, also known as White Frequency Modulation (WHFM)), Random Run PM (RRPM, or Flicker FM (FLFM)), and Random Walk (RWFM).

The crystal oscillators\` typical frequency range is $10\div100$ MHz and delay τ typical range is $0\div100$ ns. Therefore, the value τ x f_h = $0\div30$. The substitution of (9) into (5) gives us the rms jitter $J_{rms} = \sqrt{<j^2(t)>}$. The calculated results are shown in Table 1 and Table 2 for rectangular and Lorentzian shapes of h(f) functions correspondently. Cutoff frequency f_h is defined as effective frequency bandwidth $f_h = (1/h(0))\int_0^\infty h(f)df$. In the first column of Table1 the results are written similar to the results had been obtained in [19,20]. As we can see, at $2f_h\tau >>1$ the jitter value does not depend on the shape of h(f) function. At the middle value $\tau\cdot f_h$ the J_{rms} dependence on the shape of h(f) function is week, but at $2f_h\tau<<1$ (the last columns in Tables 1,2) the dependence on the h(f) shape becomes significant: for rectangle h(f) the J_{rms} is proportional to τ, but for Lorentzian h(f) the J_{rms} is

TABLE 1. JITTER RMS FOR RECTANGLE SHAPE h(f) **WITH EFFECTIVE FREQUENCY** f_h.

Spectrum type	$f_h\,\tau>>1$	$f_h\,\tau>=1$	$f_h\,\tau<<1$
WHPM	$\frac{1}{\pi\nu_0}\sqrt{b_0 f_h}$	$\frac{1}{\pi\nu_0}\sqrt{b_0 f_h(1-\frac{\sin 2\pi f_h\tau}{2\pi f_h\tau})}$	$\frac{1}{\nu_0}\sqrt{\frac{2}{3}}\tau f_h\sqrt{b_0 f_h}$
FLPM	$\frac{1}{\pi\nu_0}\sqrt{b_{-1}(C+\ln(2\pi f_h\tau)}$	$\frac{1}{\pi\nu_0}\sqrt{b_{-1}[C+\ln(2\pi f_h\tau)-Ci(2\pi f_h\tau)]}$	$\frac{1}{\nu_0}\tau f_h\sqrt{b_{-1}}$
RWPM	$\frac{1}{\nu_0}\sqrt{b_{-2}\tau}$	$\frac{1}{\pi\nu_0}\sqrt{b_{-2}\left(2\pi\tau Si(2\pi f_h\tau)-\frac{2}{f_h}\sin^2(\pi f_h\tau)\right)}$	$\frac{1}{\nu_0}\tau f_h\sqrt{\frac{2b_{-2}}{f_h}}$

TABLE 2. JITTER RMS FOR LORENTZIAN SHAPE $h(f)=f_h^2/(f_h^2+(f\pi/2)^2)$, WITH EFFECTIVE FREQUENCY f_h.

Spectrum	$f_h\,\tau >> 1$	$f_h\,\tau >= 1$	$f_h\,\tau << 1$
WHPM	$\frac{1}{\pi\nu_0}\sqrt{b_0 f_h}$	$\frac{1}{\pi\nu_0}\sqrt{b_0 f_h(1-\exp(-4f_h\tau))}$	$\frac{2}{\pi\nu_0}\sqrt{\tau f_h}\sqrt{b_0 f_h}$
FLPM	See Fig.3c	$\frac{1}{2\nu_0 f_h}\sqrt{b_{-1}[C+\ln(2f_h\tau)-Za(2f_h\tau)]}$	See Fig.3.c
RWPM	$\frac{1}{\nu_0}\sqrt{b_{-2}\tau}$	$\frac{1}{\nu_0}\sqrt{\frac{b_{-2}}{4f_h}(\exp(-4f_h\tau)+4f_h\tau-1)}$	$\frac{\tau}{\nu_0}\sqrt{2b_{-2}f_h}$

Where: $Si(x)\equiv\int_0^x \frac{\sin(z)}{z}dz$, $Ci(x)\equiv\int_\infty^x \frac{\cos z}{z}dz$, $Za(x)\equiv\frac{1}{2}(\exp(-x)Ei(x)+\exp(x)Ei(x))$,

$Ei(x)\equiv\int_\infty^x \frac{\exp(z)}{z}dz$, C=0.5772, $k=\tau f_h$.

proportional to $\sqrt{\tau}$. This dependence J_{rms} at small τ opens the way for investigation of the h(f) function and XO` phase noise behavior at high frequencies.

As obtained above, the contribution of low frequency components to the jitter value is reduced. Therefore, consider the first three components of noise (9) with β=0,-1,-2. Input the coefficients K_{WHPM}, K_{FLPM}, and K_{RWPM} characterize the contribution of each component to the total jitter value. The total jitter:

$$J_{rms}= T_0\sqrt{b_0K^2_{WHPM}+b_{-1}K^2_{FLPM}+b_{-2}K^2_{RWPM}} \qquad (10)$$

These coefficients are shown in Fig. 3a,b,c as they were obtained analytically with Mathlab. These figures are useful for engineering calculations noise component contribution to the J_{rms}. As it follows from the Fig. 3, jitter is mainly defined by white phase noise mainly. Fig.3a shows the difference of the jitter behavior at small value of τ for Lorentzian and rectangular shape of h(f). From Fig. 3a we may provide the estimation of the cutoff frequency of the oscillators` electronic circuits (the phase noise bandwidth) when the jitter dependence versus τ becomes steeper, - at $\tau \times f_h \approx 1/2$.

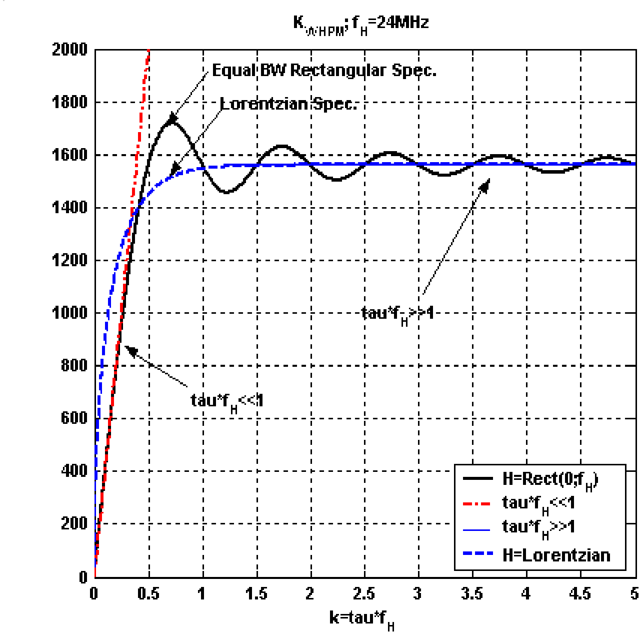
Example of the ring oscillator : In [15] the experimental results obtained for 2.8-GHz ring oscillators. Experimental results show $\sqrt{\tau}$ dependence at τ less than 30 ns and up to 1ns. This may be interpreted as evidence of Lorenyzian shape of the oscillator`s electronic circuits (h(f) transition function) with cutoff frequency $f_h \approx$ 500MHz.

Example of the XO: Consider the results of the phase noise and jitter measurements of 24 MHz XO. The level of White phase noise was -140 dBc/Hz at 2.5 KHz ÷ 100KHz; FLPM -135 dBc/Hz at 500 Hz, and RWPM -52 dBc/Hz at 1 Hz. The calculations show that FLPM and RWPM contribution to the jitter value is negligible and jitter value is defined mainly by White phase noise in the bandwidth from 2.5 KHz to f_h. We do not know the behavior of the phase noise above 100KHz and f_h. Let us assume that oscillator has constant White phase noise level b_0 at all frequencies from F_{PD}=100 KHz up to f_h. The measured XO had pulse output signals. Let us assume that f_h equals to $3\nu_0$. Notes that all supposing have done are very rough.

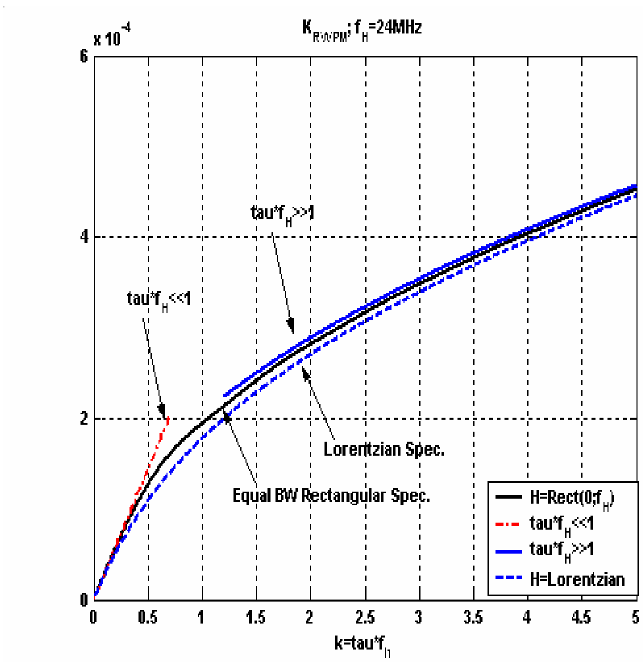
For White noise $J_{rms}=(1/\pi\nu_0)\sqrt{b_0 f_h(1-(\sin 2\pi f_h\tau)/2\pi f_h\tau)}$. The member in brackets under the square root changes from 1.2 to 0.36 and the estimation $J_{rms}=(0.35\div 0.19)\,T_0\sqrt{b_0 f_h}$ Takes place. Further calculations give us jitter`s limits 17.7÷9.6 ps, and measurements give us 10.2 ps. In spite of the rough assumptions, the measured value meets the limits of the estimation.

JITTER VARIANCE ESTIMATION

Let us define the sampling rate of the SO due the measurements. The SO` trigger circuit operates as a divider with the division coefficient M. The sampling frequency F_S results in the screen points in real time (the time of the screen commonly named " the effective time"). It will be M time



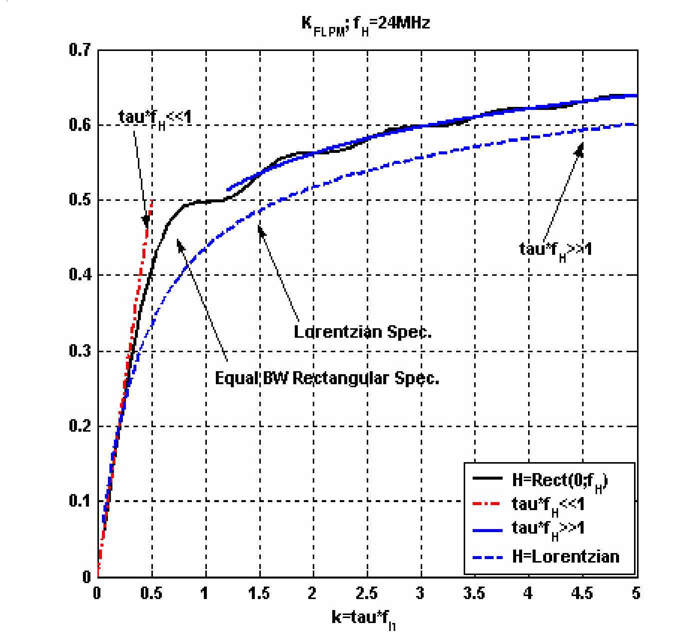
3.a) Coefficient K_{WHPM} of White noise contribution.



3.b) Coefficient K_{RWPM} of flicker noise (β=2) contribution.

Fig. 3. The coefficients of the noise contribution to the jitter.

lower than the frequency $\nu_0 = 1/T_0$ of the measured clock: $F_S = \nu_0/M$. Each measurement of the transition position is taken at crossing point U_0 (Fig.1) during one time-base ramp of the SO. Input K - is the total number



3.c) Coefficient K_{FLPM} of flicker noise (β=1) contribution.

of the points on the screen\` time base, and $T_R$ is the duration of the screen\` time base. Each time-base requires K samples of the signal, and the sampling rate of the crossing events will be K time lover than F_S: ν_0/MK. Designate N -the total number of the crossing events used for variance calculations. Using (1) and [1], we may write for jitter variance:

$$\sigma_j^2(N,T_s,\tau) = \frac{8}{(2\pi\nu_0)^2}\frac{N}{N-1}\int_{f_L}^{f_h} S_\varphi(f)\sin^2(\pi f\tau)df \qquad (11)$$

here is $f_L = \nu_0/2MKN = F_S/2KN$.

For jitter variance estimations chousing of the lowest frequency limit at the integral (11) is very important for practical calculations. For sampling frequency of the SO F_S=100KHz, K=1000, and N=100, the lowest frequency limit will be $f_L = F_S/2KN$=0.5Hz.

PART II. JITTER TIME INTERVAL MEASUREMENTS (THE SECOND JITTER DIFFERENCES)

For clock jitter characterization, the second jitter differences are implied: clock period variations, half-period variations, duty cycle variations, or cycle-to-cycle jitter [8,9,12,17-20]. All these characteristics are time interval measurements of the waveform on the oscilloscope screen. Before the consideration of the second jitter differences we have to consider SO\` unique operation. The waveform, on the SO\` screen, consists of the points\` sequence (samples). The number of points for one waveform period on the screen

$$m = \text{ent}\,(T_0 \cdot K/T_R) \qquad (12)$$

In real time one period of the signal waveform is to be measured $\theta = (m-1)\cdot T_0 \cdot M$:

$$\theta = \frac{T_0^2}{T_R}\,(K\text{-}1)\cdot M \qquad (13)$$

If the maximum sampling rate of the SO is F_S, from (13):

$$\theta = \frac{K-1}{F_S T_R}\,T_0 \qquad (14)$$

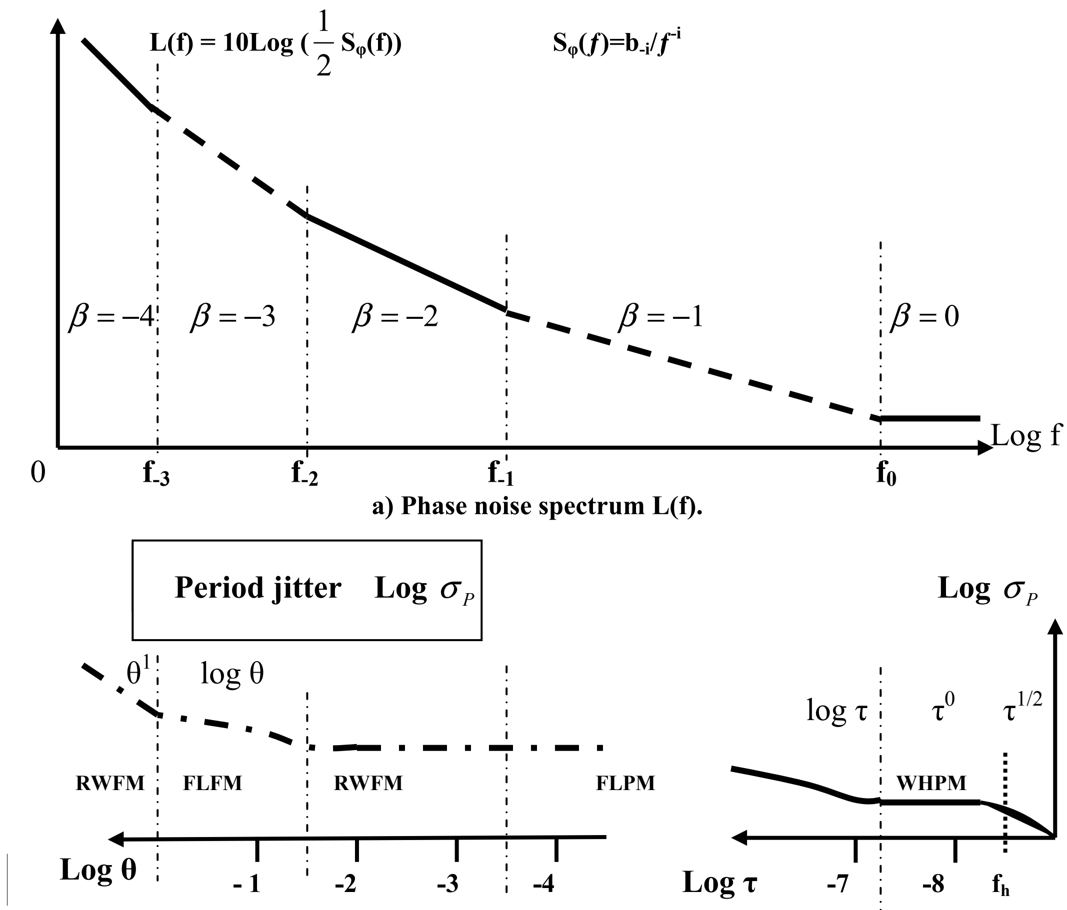
Consider the Period jitter - the min square of the period deviation referred to the average period value. From (3) we get:

$$\sigma_P^2 = \langle[(\varphi(t+\tau)-\varphi(t)) - (\varphi(t+\theta+\tau)-\varphi(t+\theta))]^2\rangle/(2\pi\nu_0)^2 \qquad (15)$$

In frequency domain:

$$\sigma_P^2 = \frac{8}{(\pi\nu_0)^2}\int_0^\infty S_\varphi(f)h(f)\sin^2(\pi f\theta)\sin^2(\pi f\tau)df \qquad (16)$$

here integrating is done from ‘0’, as the low frequency components in (16) are reduced more by two sine functions. Notice that for increasing the measurements` sensitivity we should choose pretty large delay τ, and the trigger point in this case is placed “outside” the SO screen.
Fig.4 illustrates the relations between spectrum components with different β, and Period jitter dependence versus τ (16). It illustrates that the value of τ is not higher than 1 mks, and θ not less than 100 mks. According to (14) the typical value of θ is about $1 \div 10$ Ms, usually. The calculations of the (16) may be done similar to the Tables 1,2. Must be noted that $\theta \gg \tau$ is important constraint on the sampling oscilloscope measurement method. As we can see from (16) and Fig.4, the Period jitter has two arguments with two different time scales, nanosecond and millisecond time ranges.



b) Sigma – tau diagrams.

Fig. 4. The illustration of two arguments with two time scales at second jitter differences: a) typical oscillators` phase noise spectrum; b) Period jitter sigma-tau diagrams.

For Half-period jitter the differences between the first and second half of one period are measured. The Half-period jitter represents the next order difference:

$$\sigma_{HP}^2 = \frac{32}{(\pi\nu_0)^2}\int_0^{\infty} S_\varphi(f)h(f)\sin^4(\frac{\pi f\theta}{2})\sin^2(\pi f\theta)df \qquad (17)$$

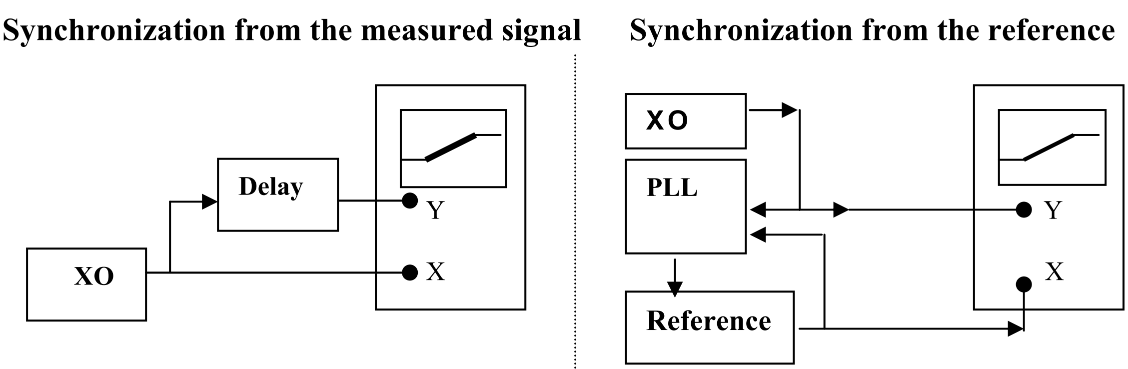
Circle-to-Circle jitter will differ from (17) by the argument value only; see Fig.5, equation (18).

Underline that $\theta >> \tau$ is important specific of the sampling oscilloscope measurement method at SO synchronization from the reference.

THE SYNCHRONIZATION AFFECT AT OSCILLATORS` PHASE NOISE TRANSFORMATION

We have investigated SO self-synchronization. The advantage of such type of the synchronization is avoiding reference devices. This is crucial for XO manufacturing and implementation. XOs have wide range of the frequencies and the creation of a large number of references often impossible. Otherwise, there are many cases when the synchronization from the reference devices is needed. In the case of SO synchronization from the reference, the first sine-function under integrals in (16)-(18) disappears and relations between jitter and phase noise is simplified, Fig.5, right column. The same equations will be in case jitter measurements are performed refer to the trigger point. Particularly, transition jitter is obtained by famous equation – phase fluctuations spectrum integration (19). As it follows from the equations (16) - (21), in synchronization from the reference, the jitter parameters (19) - (21) are defined by low frequency part of the phase noise spectrum, mainly. Otherwise, in sampling oscilloscope synchronization from the measured signal, the jitter is defined by high frequency part of the phase noise spectrum.

The equation (20) was considered in [19], and equations (20), (21) in [20]. Both papers contain detail calculations for different types of the phase fluctuations spectrum. At the implementation of the results obtained in [19, 20] the equations (13) and (14) for argument θ are useful. These equations define the time interval (in real time) during which the jitter differences measure by sampling oscilloscope.



Transition jitter

$$\sigma_j^2 = \frac{2}{(\pi\nu_0)^2}\int_{f_L}^{f_h} S_\phi(f)\sin^2(\pi f\tau)df \qquad (16) \qquad\qquad \sigma_j^2 = \frac{2}{(2\pi\nu_0)^2}\int_{f_L}^{f_h} S_\phi(f)df \qquad (19)$$

Period jitter

$$\sigma_P^2 = \frac{8}{(\pi\nu_0)^2}\int_{f_L}^{f_h} S_\phi(f)\sin^2(\pi f\theta)\sin^2(\pi f\tau)df \qquad (17) \qquad\qquad \sigma_P^2 = \frac{2}{(\pi\nu_0)^2}\int_{f_L}^{f_h} S_\phi(f)\sin^2(\pi f\theta)df \qquad (20)$$

Circle-to-Circle jitter

$$\sigma_{C-C}^2 = \frac{32}{(\pi\nu_0)^2}\int_{f_L}^{f_h} S_\phi(f)\sin^4(\pi f\theta)\sin^2(\pi f\tau)df \qquad (18) \qquad\qquad \sigma_{C-C}^2 = \frac{8}{(\pi\nu_0)^2}\int_{f_L}^{f_h} S_\phi(f)\sin^4(\pi f\theta)df \qquad (21)$$

Fig. 5. The illustration of the differences of the phase fluctuations spectrum transformation depends of the sampling oscilloscope synchronization and type of the measurements.

In [5] the equation similar (21) was obtained at jitter estimation with using the Allan variance calculations. The famous way of the timing jitter calculation is $\tau\cdot\sigma_y(\tau)$, where $\sigma_y(\tau)$ is the square root of the Allan variance, – the first difference of fractional frequency y(t) averaged over a period τ [5]:

$$\sigma_y(\tau) = \langle \tfrac{1}{2}[y(t+\tau) - y(t)]^2 \rangle^{1/2} \qquad (22)$$

Compare (13) in [5] and formula (21) we may see, that the estimation jitter by Allan variance characterizes the Circle-to-Circle jitter at measurements performed refer to the trigger point:

$$\sigma^2_{C-C} = 4\cdot\theta^2\cdot\sigma_y^2(\theta) \qquad (23)$$

The multiplier 4 in (23) comes due the ½ in the Allan variance definition (22).

THE TRANSITION JITTER AT SINUSOIDAL PHASE MODULATION

As the illustration of the differences between types of the synchronization, consider an affect of harmonic modulation at transition jitter measurements for two types of the synchronization: from the measured signal and from the reference. Assume that the oscillator` phase is modulated by sine function with amplitude Δ_φ and frequency F_0 : $\Delta_\varphi\cdot\sin(2\pi F_0 t)$. The natures of such sinusoidal phase modulation are spurs, 60 Hz harmonics, etc.. This modulation has a power $(\Delta_\varphi)^2/2$ and creates a linear spectrum components with amplitude $(\Delta_\varphi)^2/4$ symmetrical refer to the carrier.
From Fig.5, formula (19), the jitter value is obtained:

$$\sigma_j^2 = (\Delta_\varphi)^2/2(2\pi\nu_0)^2 \qquad (24)$$

In case synchronization from the measured signal, from (5) we will get the jitter value:

$$\sigma_j^2 = 2((\Delta_\varphi)^2/(2\pi\nu_0)^2)\cdot\sin^2(\pi\tau F_0) \qquad (25)$$

For harmonic frequency small enough ($F_0 \ll 1/\tau$), we may write:

$$\sigma_j^2 = (\tfrac{1}{2})(\Delta_\varphi)^2\,\tau^2\,(F_0/\nu_0)^2 \qquad (26)$$

So, jitter on harmonic modulation at self-synchronization is decreasing with decreasing τ and harmonic` frequency F_0. From (24) and (26) obtain the ratio of the jitter variances:

$$\frac{\sigma_j^2 \text{ synchr. of ref}}{\sigma_j^2 \text{ synchr. of signal}} = \frac{1}{(2\pi)^2}\frac{1}{(F_0\tau)^2} \gg 1 \qquad (27)$$

So, the type of the synchronization significantly affects jitter measurements.

THE DISCUSSION OF THE XO SPECIFICATION

So, we have many options for jitter specification and characterization depending on the type of measurements and synchronization. The question arises, what we should choose for oscillator jitter specification and characterization.
At XOs` jitter specification we have to start from the analysis of the phase noise transformation in the system where this XO has to be implemented. After that we have to choose:
a) measurement type (transition, or period jitter, etc.);
b) measurements reference (to the trigger point or not);
c) synchronization type.
We have to choose such type of the measurements and measurement conditions that would result in phase noise transformation, close to its transformation in the system where we are going to implement this XO.

In practice, for example, the jitter may be specified at low frequency spectrum, but the jitter characterization may be done with self-synchronization and will characterize the high frequency phase noise components. The measured value may be lower than specified one, but it may represent completely different jitter characteristic and real oscillator may not meet the specification.

Specifying XOs, proper characteristics and parameters have to be specified. These parameters reviewed in the present paper: type of the synchronization and measurements; delay interval τ ; the oscilloscope time base duration T_R and number of points K ; etc.

If only the jitter level specified, as commonly done, jitter is not well defined and in different measurements of one XO we will have different results.

CONCLUSION

1. The analysis of the phase noise transformation at sampling oscilloscope measurement method has presented. The new relations of the phase noise and jitter measurements obtained for transition jitter and for other jitter characteristics: Half-Period jitter, Circle-to-Circle jitter, etc.
2. For synchronization from the measured signal jitter is mainly affected by the high frequency part of the phase noise. For reference synchronization, or trigger point reference, the jitter characterizes the whole phase noise spectrum, and is mainly defined by low frequency part of the phase noise spectrum components.
3. Jitter measurements with self-synchronization open a way to investigation of the high frequency phase noise spectrum and cutoff frequency of the oscillator`s electronic circuits up to 50GHz and higher. This information cannot be obtained with any other existing methods. Jitter and phase noise

measurements may be considered as two distinct measurement methods accomplishing each other.

4. It is shown that in the self-synchronization case transition jitter with phase noise relation is defined by frequency fluctuations spectrum integral, but not the phase fluctuation spectrum integral, as often done.

5. Specifying the XOs` jitter one must refer: type of the measurements, type of the synchronization, measurement conditions.

6. It is shown that the famous way of the jitter characterization by Allan variance, characterizes the Circle-to-Circle jitter measurements performed refer to the trigger point.

7. One of the reasons for probable mistakes in jitter characterization and specification is a lack of the standards in this field of the measurements.

REFERENCES

FREQUENCY DOMAIN AND ELECTRONIC COUNTER MEASUREMENT METHOD

1. J.A. Barnes, A.R. Chi, L.S. Cutler, D.J. Healy, D.B. Leeson, T.E. McGunigal, J.A. Mullen, Jr., W.L. Smith, R.L. Sydnor, R.F.C. Vessot, G.M.R. Winkler, "Characterization of the frequency stability," IEEE Trans. Instrum. Meas., IM-20, 105-120 (1971).

2. D.B Sullivan, D.W Allan, D.A. Howe and F.L. Walls, “Characterization of Clocks and Oscillators”, *NIST Technical Note 1337*, U.S Department of Commerce, National Institute of Standards and Technology, March 1990.

3. S.R. Stein, "Frequency and Time - Their Measurement and Characterization", Chapter 12, pp.191-416, *Precision Frequency Control*, Vol. 2, Edited by E.A. Gerber and A. Ballato, Academic Press, New York, 1985.

4. C.A. Greenhall, "Frequency Stability Review", Telecommunications and Data Acquisition Progress Report 42-88, Oct-Dec 1986, Jet Propulsion Laboratory, Pasadena, CA, pp. 200-212, Feb. 1987.

5. W.F. Walls and F.L. Walls, “Computation of Time-domain Frequency Stability and Jitter from PM Noise Measurements”, Proc. 2001 IEEE Int. Freq. Cont. Symp., 161-166.

6. J.R. Vig, "Quartz Crystal Resonators and Oscillators For Frequency Control and Timing Applications - A Tutorial".

7. A. Malahov, “The fluctuations in oscillators systems” M. Nauka, Glavn. Red. Phys.-Math. Lit., 1968, 661 p. (in Russian).

SAMPLING OSCILLOSCOPE JITTER MEASUREMENT METHOD

8. B. Gudkovich, I. Zamek. “Noise and jitter influence on pulses parameters measurements”. Tech. Sredstv Svyazi. Radioizmerit. Tech., No. 5, 1976. (in Russian).

9. B. Gudkovich, I. Zamek. “The investigation of the jitter presence in sampling oscilloscopes and pulse generators”. Tech. Sredstv Svyazi. Radioizmerit. Tech., No. 5, 1977. (in Russian).

10. B. Gudkovich, I. Zamek. “The way of the jitter` spectrum measurements”. Tech. Sredstv Svyazi. Radioizmerit. Tech., No. 1, 1979. (in Russian).

11. I. Zamek. “The signals and noises transformation in sampling oscilloscopes”. Tech. Sredstv Svyazi. Radioizmerit. Tech., No. 1, 1981. (in Russian).

12. D. Chandler, “Phase jitter - phase noise and voltage controlled crystal oscillators”, Corning frequency control Inc. Technical Note.

13. Joe Adler, “Clock-source jitter: A clear understanding aids oscillator selection”, EDN, Feb. 1999.

14. J. McNeill, “Jitter in ring oscillators”, IEEE J. Solid-State Circuits, vol. 32, 870-879, June 1997.

15. A. Hajimiri, S. Lymotyrakis, and T. H. Lee, “Jitter and Phase Noise in Ring Oscillators,” IEEE Journal of Solid-State Circuits, pp.790-804, June 1999.

16. J. McNeill, “ A Simple Method for Relating Time- and Frequency-Domain Measures of Oscillator Performance”, Proceedings of the 2001 IEEE Southwest Symposium on Mixed Signal Design (SSMSD2001), Austin, TX, February, 2001.

17. M. Shimanouchi, “An approach to consistent jitter modeling for various jitter aspects and measurement methods”, Proceedings of the 2001 International Test Conference, pp. 848-857.

18. R. Poore “Phase noise and jitter”, Agilent EEsof EDA, Agilent Technologies Inc., 2001.

19. B. Drakhlis, “Calculate oscillator jitter by using phase-noise analysis”, MICROVAVE & RF, Part1, January 2001; Part 2, February 2001.

20. D.A. Howe, “Clock Jitter estimation from PM Noise Measurements”, Proc. MTT-ARFTG Conf., April, 2002.

STANDARDS AND RECOMMENDATIONS

21. International Telecommunication Union (ITU-T) Recommendation G.810. “Definition and terminology for synchronization networks”, August 1996.

22. MIL-PRF-55310, Oscillators, Crystal, General Specification For.